

MATHEMATICS HSSC-II

Time allowed: 2:35 Hours

Total Marks Sections B and C: 80

SECTION – B (Marks 48)

Q. 2 Solve the following Questions.

(12 x 4 = 48)

(Use of graph paper is not necessary. Candidates can make their own grid on answer book)

(i)	Given that $f(x) = x^3 - px^2 + qx + 1$ if $f(3) = -5$ and $f(-2) = 5$ then find the values of p and q	04	OR	Show that $\frac{dy}{dx} = \frac{y}{x}$ if $\frac{x}{y} = \tan^{-1}\left(\frac{x}{y}\right)$	04
(ii)	Find the derivative of $x^{\frac{3}{4}}$ by using definition w.r.t. x . Also calculate the derivative at $x = 16$	04	OR	For the given function $f(x) = \begin{cases} 2ax + 3 & \text{if } x < 2 \\ 2x + 2a & \text{if } x \geq 2 \end{cases}$ Find the value of a that $\lim_{x \rightarrow 2} f(x)$ exists	04
(iii)	Evaluate $\int_0^{\frac{\pi}{2}} \cos^3 x \sqrt{\sin x} \, dx$	04	OR	Evaluate the limit. $\lim_{\theta \rightarrow 0} \frac{1 - \cos 3\theta}{1 - \cos 5\theta}$	04
(iv)	Find the point on the curve $y = x^2 + 2$ that is closest to the point $(18, 2)$	04	OR	Evaluate $\int \ln(x + \sqrt{x^2 + 1}) \, dx$ (by parts)	04
(v)	Find $\frac{dy}{dx}$ if $y = \ln \left[\frac{x(x^2 - 3)^2}{(x^2 - 4)^{\frac{1}{2}}} \right]$	04	OR	Find the area above the x -axis bounded by the curve $y^2 = 3 - x$ from $x = -1$ to $x = 2$	04
(vi)	Find the equation of an ellipse with vertices $(-1, 1)$, $(5, 1)$ and foci $(4, 1)$, $(0, 1)$	04	OR	If two vertices of an equilateral triangle are $A(-4, 0)$ and $B(4, 0)$. Find the third vertex. How many of these triangles are possible?	04
(vii)	Graph the feasible region subject to the following constraints $2x + 8y \leq 60$, $4x + 4y \leq 60$, $x \geq 0$, $y \geq 0$ Also find the corner points.	04	OR	Find a joint equation of the straight line through the origin represented by $2x^2 - 5xy + 3y^2 = 0$	04
(viii)	Find the equation of a straight line through intersection of the lines $x + 2y + 3 = 0$, $3x + 4y + 7 = 0$ and making equal intercept on axes.	04	OR	Graph the feasible region subject to the following constraints $3x - y \geq -1$, $x + y \leq 5$, $x \geq 0$, $y \geq 0$ Also find the corner points.	04
(ix)	In any triangle ABC , Prove that $c^2 = a^2 + b^2 - 2ab \cos C$ (Use vector method)	04	OR	Write the equation of parabola with given elements directrix $y = 3$, vertex $(2, 2)$	04
(x)	Solve the differential equation $y - x \frac{dy}{dx} = 3 \left(1 + x \frac{dy}{dx} \right)$ Also find the particular solution if $y = 7$ and $x = 16$	04	OR	Find the equation of the normal to the parabola $y^2 = 8x$ which is parallel to the line $2x + 3y = 10$	04
(xi)	Find the equation of hyperbola with center $(2, 2)$ horizontal transverse axis of length 6 and eccentricity $e = 2$	04	OR	Find a unit vector perpendicular to the plane containing \underline{a} and \underline{b} . Also find the sine of the angle between them, $\underline{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\underline{b} = -\hat{i} - \hat{j} - \hat{k}$	04
(xii)	Write the equation of tangent line to the circle $x^2 + y^2 + 4x + 2y = 0$ drawn from $(-1, 2)$. Also find the tangential distance.	04	OR	Find the volume of tetrahedron whose vertices are $A(2, 1, 8)$, $B(3, 2, 9)$, $C(2, 1, 4)$ and $D(3, 3, 10)$	04

SECTION – C (Marks 32)

Note: Solve the following Questions.

(4 x 8 = 32)

(Use of graph paper is not necessary. Candidates can make their own grid on answer book)

Q.3	Let $f(x) = \begin{cases} lx + 2m & \text{if } 0 \leq x < 1 \\ mx + 2 & \text{if } 1 \leq x < 2 \\ 3lx + 4 & \text{if } 2 \leq x < 4 \end{cases}$	a. Find the values of l and m such that function f is continuous at $x = 1$ and $x = 2$ b. After finding values of l and m , sketch the graph of $f(x)$	08	OR	Discuss the function defined as $f(x) = \sin x + \frac{1}{2\sqrt{2}} \cos x$ for extreme values in the interval $(0, 2\pi)$	08
Q.4	Find the centre, foci, eccentricity, vertices and directrices of the conic $9x^2 - 18x + 4y^2 + 8y - 23 = 0$		08	OR	Show that $\int e^{ax} \sin bx dx = \frac{1}{\sqrt{a^2 + b^2}} \sin \left(bx - \tan^{-1} \left(\frac{b}{a} \right) \right) + c$ By using this formula evaluate $\int e^{2x} \sin 3x dx$	08
Q.5	Find the distance between the parallel lines: $12x + 5y - 6 = 0, 12x + 5y + 13 = 0$ a. Sketch the lines. b. Find the equation of parallel lines lying mid way between them. c. Write the equation of line in two intercept form.		08	OR	A toy factory manufactures two types of toys: remote car and scooty by using two machines A and B. Machine A has at most 110 hours available and machine B has a maximum of 300 hours available. Manufacturing a remote car requires 5 hours in machine A and 10 hours in machine B. While manufacturing of scooty requires 30 hours in machine A and 20 hours in machine B. If the factory gets profit of Rs. 300 per remote car and Rs. 400 per scooty, how many both toys should be manufactured to get maximum profit?	08
Q.6	Evaluate $\int \frac{2x^2 - x - 7}{(x+3)(x-2)(x^2 - 2x + 3)} dx$		08	OR	Find the equation of tangent and normal at a point $\left(5, \frac{16}{9} \right)$ to the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$. Also find the value of c , when the line $y = -x + c$ will touch the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$	08

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